a reciprocal curve are then obtained, but the line of proof cannot be included in this abstract.

In the latter part of the paper, the differential equations of the quadriquadric curve are integrated to obtain the absolute invariant of the curve, and the conditions to be satisfied by the four excubo-quartics having closest contact with a curve at a point are found.

The equations to a quadriquadric curve referred to the canonical axes at a point of the curve, are

$$\left.\begin{aligned} u &= y-x^2-p_3(xz-y^2) = 0, \\ v &= z-xy-p_1z^2-p_2(xz-y^2) = 0, \end{aligned}\right\}$$

where $p_1 = \alpha_6$, $p_2 = \alpha_7/\alpha_6$, $p_3 = \beta_7/\alpha_6$.

These represent two out of the family of quadrics which contain the quadriquadric; the quadric represented by $v = 0$ is that which touches at the origin the osculating plane to the quadriquadric at the origin. If we call the fourth point, at which that osculating plane meets the curve, the tangential of the origin, the quadric represented by $u = 0$ is that quadric of the family which touches, at the tangential, the osculating plane at the tangential.

These two quadrics are called the canonical quadrics at the point, and it is shown how to find the equations to the canonical quadrics at any point and how to express the canonical invariants at any point in terms of those at the origin. The relations between the invariants at a point and its tangential are found, and lead to the discussion of the singular points indicated by $p_1-p_2p_3 = 0$ and $p_1^2-p_1p_2p_3-p_3^3 = 0$.

XI. "On the Singular Solutions of Simultaneous Ordinary Differential Equations and the Theory of Congruencies." By A. C. DIXON, M.A., Fellow of Trinity College, Cambridge, Professor of Mathematics in Queen's College, Galway. Communicated by J. W. L. GLAISHER, Sc.D., F.R.S. Received June 7, 1894.

(Abstract.)

§ 1. This paper is an attempt to shew how the singular solutions of simultaneous ordinary differential equations are to be found either from a complete primitive or from the differential equations.

The *latter* question has been treated by Mayer ('Math. Ann.,' vol. 22, p. 368) in a somewhat different way, but with the same result. He also gives a reference to a paper in Polish by Zajączkowski (summarised in vol. 9 of the 'Jahrbuch der Fortschritte der Mathematik'), and to one by Serret in vol. 18 of 'Liouville's Journal.'

The general result is that there may be as many forms of solution as there are variables (the differential equations being of the first order, to which they may always be reduced). Each form is derived from the one before by the process of finding the envelope, and each contains fewer arbitrary constants by one than the form from which it is directly derived.

The general theory is given in § 2 for the case when the differential coefficients are given explicitly in terms of the variables. In § 3 it is extended to the case when they are given implicitly, and in § 4 it is shown how the singular solutions are to be formed from the differential equations themselves. In §§ 5—9 the theory is connected with that of consecutive solutions belonging to the complete primitive. §§ 10—13 are taken up with geometrical interpretations relating to plane curves, and also to curves in space of $n+1$ dimensions, $n+1$ being the number of variables. In §§ 14—16 the case is discussed in which a system of singular solutions is included in a former system or in the complete primitive.

The rest of the paper contains the application of the theory to certain examples. The first example (§§ 17—21) is the case of the "lines in two osculating planes" of a twisted curve, and in particular of a twisted cubic. The particular example is given by Mayer and Serret. The second (§§ 22—26) is that of the congruency of common tangents to two quadric surfaces, and generally (§§ 27—38) of the bitangents to any surface. The third (§§ 39—49) is that of the essentially different kind of congruency which consists of the inflexional tangents to a surface. It seems natural to call these two kinds of congruency *bitangential* and *inflexional* respectively. The fourth example (§§ 50—52) is that of a system of conics touching six planes. The fifth (§§ 53—60) is that of a doubly infinite system of parabolas in one plane, the differential equation being a case of an extension of Clairaut's form $y = px + f(p)$, which is explained in §§ 53—55.

XII. "The Spectrum Changes in β Lyræ. Preliminary Note." By J. Norman Lockyer, C.B., F.R.S. Received June 13, 1894.

The spectrum of this well known variable star was first investigated photographically by Professor Pickering, at Harvard College Observatory, and a preliminary account of the results was published in 1891.* Dark and bright lines were found to be associated in the spectrum, and further, the bright lines were found to change their positions with respect to the corresponding dark ones according to the interval of time which had elapsed since the preceding minimum.

* 'Ast. Nach.,' 2707; 'Observatory,' 1891, p. 341.